

Description

Below are listed the longest prime siteswaps for a variety of cases, organized by number of balls and maximum throw value. Throws greater than 9 are listed as A, B, C, and so on. For an explanation and summary of these results please refer to my [prime siteswaps page](#).

The calculation times listed below are from a variety of different machines and algorithms (the code was evolving as I was doing this). They are for entertainment purposes only.

2 Balls

2 Balls, Height 3

$(L, L_{\text{bound}}, \text{States}) = (3, 3, 3)$

330

2 Balls, Height 4

$(L, L_{\text{bound}}, \text{States}) = (4, 4, 6)$

4130
4400

2 Balls, Height 5

$(L, L_{\text{bound}}, \text{States}) = (8, 8, 10)$

Complete: (1 pattern)

+2-+-4--

2 Balls, Height 6

$(L, L_{\text{bound}}, \text{States}) = (12, 12, 15)$

Complete: (1 pattern)

+2-5---5---

2 Balls, Height 7

$(L, L_{\text{bound}}, \text{States}) = (18, 18, 21)$

Complete: (1 pattern)

+3---6----6-----

2 Balls, Height 8

$(L, L_{\text{bound}}, \text{States}) = (24, 24, 28)$

Complete: (1 pattern)

+3--7---+--7-----+7-----

2 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (32, 32, 36)$

Complete: (1 pattern)

+4---+---8---+---8-----+8-----

2 Balls, Height 10

$(L, L_{\text{bound}}, \text{States}) = (40, 40, 45)$

Complete: (1 pattern)

+4---9---+---9---+---9-----+9-----

2 Balls, Height 11

$(L, L_{\text{bound}}, \text{States}) = (50, 50, 55)$

Complete: (1 pattern)

+5---+---A---+---A---+---A---+---A-----

2 Balls, Height 12

$(L, L_{\text{bound}}, \text{States}) = (60, 60, 66)$

Complete: (1 pattern)

+5----B-----+----B-----+----B-----+----B-----+B-----

3 Balls

3 Balls, Height 4

$(L, L_{\text{bound}}, \text{States}) = (4, 4, 4)$

4440

3 Balls, Height 5

$(L, L_{\text{bound}}, \text{States}) = (8, 8, 10)$

Complete: (1 pattern)

++1+--+3-

3 Balls, Height 6

$(L, L_{\text{bound}}, \text{States}) = (15, 16, 20)$

Type I Incomplete: (6 patterns)

```
++2-+-+2-+-+5--  
++2-+-+11+-+5--  
++1-+-+4-+-+4--  
++1-+-+4-+-+13-  
++1-+-55-+-+4--  
+35-+-+2-+-+5--
```

Type II Incomplete: (0 patterns)

3 Balls, Height 7

$(L, L_{\text{bound}}, \text{States}) = (30, 30, 35)$

Complete: (1 pattern)

```
++3-+-+1+-+6-+-+4-+-+6---
```

3 Balls, Height 8

$(L, L_{\text{bound}}, \text{States}) = (49, 49, 56)$

Complete: (3 patterns)

```
++2-+-+5-+-+7-+-+2-+-+7-+-+5-+-+7-+-+  
++2-+-+6-+-+2-+-+7-+-+7-+-+4-+-+7-+-+
```

++1+-----+6+-----+6+-----+2+-----+7+-----+7+-----+6+-----

3 Balls, Height 9

(L, L_{bound}, States) = (74, 74, 84)

Complete: (1 pattern)

++5+-----+1+-----+8+-----+7+-----+1+-----+8-8+-----+8+-----+6+-----+8+-----

3 Balls, Height 10

(L, L_{bound}, States) = (108, 108, 120)

Complete: (0 patterns)

++3+-----+6+-----+9+-----+9+-----+4+-----+9+-----+2+-----+9+-----+9+-----+6+-----+9+-----+9+-----

3 Balls, Height 11

(L, L_{bound}, States) = (149, 150, 165)

Type I Incomplete: (18 patterns)

++7+-----+4+-----+A+-----+3+-----+A+-----+A+-----+5+-----+A+-----+2+-----+A+-----+6
++6+-----+2+-----+A+-----+A+-----+7+-----+A+-----+A+-----+5+-----+2+-----+A+-----+A+-----+5
++5+-----+3+-----+A+-----+A+-----+A+-----+8+-----+1+-----+9+-----+9+-----+3+-----+A+-----+A
++5+-----+3+-----+A+-----+A+-----+A+-----+6+-----+A+-----+2+-----+9+-----+3+-----+A+-----+A

```

++4-----+1+-----A+-----9-----+A-----+8-----++A-----+2+-----+A+-----+7-----+A-----+A
++4-----+1+-----9+-----+A-----+A-----+8-----++A-----+2+-----+A+-----+7-----+A-----+A
++3-----+9+-----+9+-----+4+-----+A-----+A-----+A-----+4+-----+2+-----+A-----+A-----+A
++3-----+9+-----+9+-----+3+-----+A-----+A-----+A-----+8-----+1+-----+5+-----+A-----+A
++3-----+8+-----+A-----+3+-----+A-----+A-----+8-----+1+-----+8+-----+A-----+7
++3-----+8+-----+2+-----+A-----+A-----+A-----+5+-----+A-----+A-----+3+-----+A-----+9
++2+-----+9+-----+9+-----+9+-----+4+-----+A-----+A-----+A-----+4+-----+2+-----+A-----+A
++2+-----+9+-----+8+-----+A-----+3+-----+A-----+A-----+A-----+9+-----+3+-----+A-----+
++2+-----+9+-----+8+-----+A-----+3+-----+9+-----+A-----+2+-----+A-----+A-----+7-----+A
++2+-----+8+-----+A-----+9+-----+8+-----+A-----+2+-----+A-----+8+-----+2+-----+A-----+A
++2+-----+5+-----+A-----+9+-----+A-----+2+-----+A-----+9+-----+9+-----+3+-----+A-----+A
++1+-----+9+-----+9+-----+8+-----+A-----+3+-----+A-----+A-----+A-----+9+-----+3+-----+
++1+-----+9+-----+9+-----+8+-----+A-----+3+-----+9+-----+A-----+2+-----+A-----+A-----+7
++1+-----+7+-----+A-----+A-----+7+-----+A-----+A-----+5+-----+A-----+3+-----+A-----+

```

Type II Incomplete: (0 patterns)

3 Balls, Height 12

(L, L_{bound}, States) = (200, 201, 220)

Type I Incomplete: (28 patterns)

```

++8-----+3+-----+B+-----+8+-----+B+-----+B+-----+6+-----+B+-----+3+-----+B+-----+2
++8-----+2+-----+B+-----+9+-----+B+-----+2+-----+B+-----+B+-----+B+-----+9+-----+B
++4+-----+8+-----+B+-----+B+-----+9+-----+1+-----+A+-----+9+-----+B+-----+2+-----+B
++4+-----+8+-----+B+-----+B+-----+7+-----+B+-----+2+-----+9+-----+B+-----+2+-----+B
++4+-----+1+-----+B+-----+B+-----+B+-----+B+-----+A+-----+5+-----+B+-----+5+-----+
++4+-----+1+-----+A+-----+B+-----+B+-----+B+-----+4+-----+B+-----+B+-----+B+-----+6+-----+
++3+-----+A+-----+A+-----+4+-----+B+-----+B+-----+A+-----+1+-----+B+-----+7+-----+B+-----+
++3+-----+A+-----+7+-----+B+-----+B+-----+B+-----+4+-----+B+-----+A+-----+3+-----+B+-----+
++3+-----+A+-----+7+-----+B+-----+B+-----+B+-----+4+-----+B+-----+2+-----+B+-----+B+-----+
++3+-----+9+-----+B+-----+8+-----+B+-----+B+-----+6+-----+B+-----+2+-----+A+-----+1
++3+-----+9+-----+B+-----+2+-----+B+-----+B+-----+B+-----+B+-----+A+-----+6+-----+B+-----+
++3+-----+9+-----+7+-----+B+-----+B+-----+6+-----+B+-----+2+-----+B+-----+B+-----+A
++3+-----+6+-----+B+-----+B+-----+B+-----+B+-----+6+-----+B+-----+2+-----+A+-----+A
++3+-----+6+-----+A+-----+B+-----+B+-----+6+-----+B+-----+2+-----+B+-----+B+-----+A
++2+-----+A+-----+A+-----+8+-----+B+-----+B+-----+A+-----+4+-----+B+-----+2+-----+

```

```

++2-+-----+A--+-----+A--+-----+8-----+--+--B-----+--+--B-----+--+3-----+--+--B-----+--+--B-----+--+A-----+--+
++2-+-----+A--+-----+A--+-----+7-----+--+--B-----+--+--B-----+--+B-----+--+4-----+--+--B-----+--+A-----+--+
++2-+-----+A--+-----+A--+-----+7-----+--+--B-----+--+--B-----+--+B-----+--+4-----+--+--B-----+--+2-----+--+
++2-+-----+A--+-----+9-----+--+--B-----+--+--2-----+--+--B-----+--+--B-----+--+--B-----+--+--B-----+--+A-----+--+6-----+--+
++2-+-----+9-----+--+--B-----+--+--8-----+--+--B-----+--+--B-----+--+A-----+--+2-----+--+--B-----+--+9-----+--+
++2-+-----+9-----+--+--B-----+--+--8-----+--+--B-----+--+--B-----+--+6-----+--+--B-----+--+--B-----+--+4-----+--+
++2-+-----+9-----+--+--B-----+--+--8-----+--+--A-----+--+1-----+--+--B-----+--+--B-----+--+9-----+--+--B-----+--+2-----+--+
++2-+-----+7-----B-----+--+--B-----+--+--B-----+--+--4-----+--+--B-----+--+--B-----+--+--B-----+--+9-----+--+2-----+--+
++1+-----+A--+-----+A--+-----+A--+-----+8-----+--+--B-----+--+--B-----+--+A-----+--+4-----+--+--B-----+--+
++1+-----+A--+-----+A--+-----+A--+-----+8-----+--+--B-----+--+--B-----+--+3-----+--+--B-----+--+--B-----+--+
++1+-----+A--+-----+9-----+--+--B-----+--+--8-----+--+--B-----+--+--B-----+--+A-----+--+2-----+--+--B-----+--+
++1+-----+A--+-----+9-----+--+--B-----+--+--9-----+--+--8-----+--+--A-----+--+1-----+--+--B-----+--+--B-----+--+9-----+--+B-----+--+
++1+-----+9-----+--+--B-----+--+--9-----+--+--B-----+--+--2-----+--+--B-----+--+--B-----+--+--B-----+--+--B-----+--+9-----+--+

```

Type II Incomplete: (2 patterns)

```

++3-+-----+A--+-----+7-----+--+--B-----+--+--B-----+--+B-----+--+4-----+--+--B-----+--+3-----+--+B-----+--+B-----+--+
++2-+-----+A--+-----+A--+-----+7-----+--+--B-----+--+--B-----+--+B-----+--+4-----+--+--B-----+--+3-----+--+B-----+--+

```

3 Balls, Height 13

(L, L_{bound}, States) = (263, 264, 286)

Type I Incomplete: (4 patterns, 45 minutes)

```

++8-----+--+3-----+--+C-----+--+C-----+--+9-----+--+C-----+--+6-----+--+C-----+--+C-----+--+3
++5-----+--+1-----+--+C-----+--+C-----+--+B-----+--+C-----+--+9-----+--+C-----+--+C-----+--+3
++5-----+--+1-----+--+B-----+--+C-----+--+C-----+--+C-----+--+9-----+--+C-----+--+C-----+--+3
++2-+-----+9-----+--+C-----+--+C-----+--+9-----+--+C-----+--+C-----+--+7-----+--+C-----+--+

```

Type II Incomplete: (4 patterns, 6.6 hours)

```

++3-+-----+B--+-----+B--+-----+3-----+--+C-----+--+C-----+--+C-----+--+5-----+--+C-----+--+C-----+--+
++2-+-----+B--+-----+B--+-----+B--+-----+3-----+--+C-----+--+C-----+--+C-----+--+5-----+--+C-----+--+
++2-+-----+B--+-----+B--+-----+9-----+--+C-----+--+C-----+--+3-----+--+C-----+--+C-----+--+
++1+-----+B--+-----+B--+-----+B--+-----+9-----+--+C-----+--+C-----+--+3-----+--+C-----+--+

```

3 Balls, Height 14

$(L, L_{\text{bound}}, \text{States}) = (337, 338, 364)$

Type I Incomplete: (38 patterns, 2 seconds)

inverses of:

```
D313131A111131B131A11111520
D22231A111112A131A111171130
D23131A111112C13131A1111330
C51115111A15111811A11171201
C3131A11111611A131A11116201
C23131A11112611B131A1111401
C22411A11112A115111A1511401
C2231A111112A131A1111711501
C3131A111112C13131A11113501
B3131A11112611B131A11116011
B2411A11112A115111A15116011
5911A11117111A1511B01161111
6B1115111A1511181B011171111
A41B011117111A15111A1125111
A2611119129111191B011116111
92912011B161111911A11116111
4B131A1111161B0131A11116111
A26111A01A11111281A11116111
B315111A112411B141B01111411
B22611119129111191B01111411
92911201C1261111911A1111411
B226111A01A11111281A1111411
4C13131A11113C0131A11111511
92911201C125111A11191111511
B2411A2011B1115111A15111511
92911120D1225111A1119111131
C22411A2011B1115111A1511131
C2231A12011B1131A1111711141
C3131A11120D113131A11113141
A61111911A1111611A029111112
B261111911A111141A029111112
```

B25111A1119111151A029111112
C225111A111911113A029111112
A411A111141C0315111A1151112
D22231A12011B1131A111171112
A31A1111151C03131A111171112
A31A111113D023131A111171112
D23131A11120D113131A1111312

Type II Incomplete: (0 patterns, 20 seconds)

3 Balls, Height 15

$(L, L_{\text{bound}}, \text{States}) = (424, 424, 455)$

Complete: (1 pattern, 1 second)

inverse of:

B411411C1151117111A1D111115111

3 Balls, Height 16

$(L, L_{\text{bound}}, \text{States}) = (524, 525, 560)$

Type I Incomplete: (20 patterns, 43 seconds)

inverses of:

D51115111C1151117111C15111C111162011
6C1115111C1151180111C15111C111161111
D3131B111112D16111141C0131B11111411
D3131B111201E1161111411B131B11111411

```

A2A111201E12261111A111C1111A11111511
B22A111201E1261111A1111B111A11111511
D3131D120111411D1315111B111A11111511
D3131D201111411D1315111B111A11111511
D2231D12011181111B15111B111A11111511
D2231D20111181111B15111B111A11111511
A2A111120F122261111A111C1111A1111131
B22A111120F12261111A1111B111A1111131
E23131D120111411D1315111B111A1111131
E23131D201111411D1315111B111A1111131
E22231D12011181111B15111B111A1111131
E22231D20111181111B15111B111A1111131
D2261111A111C1111A1111151B02A111112
E22261111A111C1111A111113B02A111112
D261111A1111B111A1111151C022A111112
E2261111A1111B111A111113C022A111112

```

Type II Incomplete: (10 patterns, 7.9 minutes)

```

++1+-----+E--+-----+E--+-----+E--+-----+E--+-----+B-----+-----+-----F-----+-----+-----F-----
++2-+-----+E--+-----+E--+-----+E--+-----+B-----+-----+-----F-----+-----+-----F-----+-----+-----F-----
++2-+-----+E--+-----+E--+-----+D-----+-----+-----F-----+-----+-----C-----+-----+-----F-----+-----+-----F-----
++2-+-----+D--+-----+-----F--+-----+D--+-----+-----F--+-----+-----C-----+-----+-----F--+-----+-----+-----F-----
++2-+-----+E--+-----+E--+-----+E--+-----+B-----+-----+-----F-----+-----+-----F-----+-----+-----F-----
++3-+-----+E--+-----+D--+-----+-----F--+-----+-----C--+-----+-----F--+-----+-----+-----F--+-----+-----+4-----+
++3-+-----+E--+-----+E--+-----+B-----+-----+-----F--+-----+-----+-----F--+-----+-----+-----F--+-----+-----+4-----+
++3-+-----+E--+-----+E--+-----+E--+-----+9-----+-----+-----F--+-----+-----+-----F--+-----+-----+-----F--+-----
++3-+-----+D--+-----+-----F--+-----+D--+-----+-----+-----F--+-----+-----+4-----+-----+-----F--+-----+-----+-----F--+-----
++4-+-----+E--+-----+E--+-----+9-----+-----+-----F--+-----+-----+-----F--+-----+-----+-----F--+-----+-----+-----F--+-----

```

3 Balls, Height 17

$(L, L_{\text{bound}}, \text{States}) = (639, 640, 680)$

Type I Incomplete: (34 patterns, 3.5 minutes)

inverses of:

D411D1111117111D14115111D11411C120115111
D411611112D1411C111111711D01411C111115111
D4112C1111117111D14160111D11411C111115111
D41162011E11411C1111117111C1411C111115111
D313D01111117111D1312B111111A11C111115111
D911111112C131C111111711E013131C111115111
D91120111D1131C1111117111D13131C111115111
D3131C111111711E0131C111111291C111115111
D2312B111111A11C1111181111C13D0111116111
D241161111C1411C1201191111C11D1111116111
B411C1111191111C11712011E12411C111116111
D411720111D12411C1111191111B11C111116111
D231C111111291C111118111D0131C111116111
D241701111C1411C1111191111C112C111116111
E31312B111111A11C11111411E1313D011111411
E22312B111111A11C1111181111C13D011111411
E314115111D11411C12011411E1411D111111411
E2241161111C1411C1201191111C11D111111411
B411C1111191111C11711201F122411C11111411
E2411720111D12411C1111191111B11C11111411
E2231C111111291C111118111D0131C11111411
E313131C11111411E19112011D1131C11111411
E3131C111111291C1111141F013131C11111411
E313131C1111141F0191111112C131C11111411
E314160111D11411C11111411E14112C11111411
E2241701111C1411C1111191111C112C11111411
F2222411C1111191111B11C120111D1117111141
F2222411C111119111C011C111112C117111141
G22222411C1111191111B11C120111D111711112
C2411C1111191111B11C11111611E0411711112
C2411C1111191111B11C1111141F02411711112
D2411C11111611C0411C1111191111C11711112
E22411C1111141C0411C1111191111C11711112
G22222411C111119111C011C111112C11711112

Type II Incomplete: (4 patterns, 43 minutes)

++3--+-+-----+F--+-+-----+F--+-+-----+E--+-+-----+G--+-+-----+4--+-+---G-----+-----+G-----
++3--+-+-----+E--+-+-----+G--+-+-----+E--+-+-----+G--+-+-----+4--+-+---G-----+-----+G-----
++4--+-+-----+F--+-+-----+E--+-+-----+G--+-+-----+4--+-+---G-----+-----+G-----+-----+G-----
++4--+-+-----+E--+-+-----+G--+-+-----+E--+-+-----+G--+-+-----+6--+-+---+-----G-----+-----+-----

3 Balls, Height 18

(L, L_{bound}, States) = (769, 770, 816)

Type I Incomplete: (50 patterns, 20 minutes)

inverses of:

```
E3161111A1111C11161111C1711111C11F1111111820111
E41161111C11161111D161111911111C11F1111111820111
E2411F1111111291111D1161111C11161111D1611160111
E231711111C11F1111112A1111C11161111D1611160111
D261111A1111C1112911111C1711111C11F111111701111
D411F1111111291111D1161111C11161111D16111801111
D31711111C11F11111112A1111C11161111D16111801111
C61111A1111C1112911111C1711111C11F1111119011111
C6111191111C11711111C1G20111111B11111C111711111
C6111A01111C11711111C1G11111112A1111C111711111
5D1161111C11161111D1611119111D011F1111111811111
4D161111A1111C1116111D01711111C11F1111111811111
D51161111D16111191111C115111E201G1111111161111
D411611112D161111A1111C1116111D01G1111111161111
D41125111E1161111A1111C11701111C1G1111111161111
D3161111C111B111112D1611118111C01F111111161111
D51161111D1611119111E011611112C11F111111161111
D3161111C111B120111E116111181111B1F111111161111
D51161111D16111191111C160111E1112F111111161111
C6111191111D1161111D111720111E15111E11111161111
D511701111D16111191111D1161111D1112E11111161111
D411F1111111A111112C161111C1116111E016111171111
D31711111C112E111111B11111C11701111D16111171111
E2411F1111111A111112C161111C1116111E01611115111
E41161111D111611112E15111E1111111511E01611115111
E411611112C11F11111115111E15111511F011611115111
E41161111D1112E1111115111E151160111E11611115111
E231711111C112E111111B11111C11701111D1611115111
E41161111D11162011F115111E1111115111D1611115111
E51161111D1161111C111E111111920111E12611115111
F22228111111B1F1111112B111B01111C111B111111511
F22228111111B1F2011111C11111A1111C111B111111511
F222611112D111711111C16111D0111E111111A11111511
```

```

F22225111E1111711111C701111C111E111111A11111511
G222228111111B1F1111112B111B01111C111B11111131
G222228111111B1F2011111C11111A1111C111B11111131
G2222611112D111711111C16111D0111E111111A1111131
G222225111E1111711111C701111C111E111111A1111131
G22222711111C11F11111112B111B01111C111A11111141
G22222711111C11F2011111C11111A1111C111A11111141
G2222611112D11161111D116111D0111E11111191111141
G222225111E111161111D1701111C111E11111191111141
H22222711111C11F11111112B111B01111C111A1111112
H22222711111C11F2011111C11111A1111C111A1111112
B711111C11F111111181111D061111A1111C111A1111112
B711111C11F111111161111E0261111A1111C111A1111112
D5111E1111116111D06111191111D1161111D117111112
H22222611112D11161111D116111D0111E1111119111112
H222225111E111161111D1701111C111E1111119111112
D261111511F0511161111D1161111C111E1111119111112

```

Type II Incomplete: (7 patterns, 4.4 hours)

```

++1+-----+G+-----+G+-----+G+-----+G+-----+G+-----+B+-----+
++2+-----+G+-----+G+-----+G+-----+G+-----+B+-----+H+-----+
++2+-----+G+-----+G+-----+G+-----+G+-----+B+-----+H+-----+
++3+-----+G+-----+G+-----+G+-----+B+-----+H+-----+H+-----+
++3+-----+G+-----+F+-----+H+-----+E+-----+H+-----+H+-----+
++4+-----+F+-----+H+-----+E+-----+H+-----+H+-----+4+-----+
++5+-----+E+-----+H+-----+H+-----+B+-----+H+-----+C+-----+

```

3 Balls, Height 19

(L, L_{bound}, States) = (917, 918, 969)

Type I Incomplete: (0 patterns)

Type II Incomplete: (4 patterns, 21 hours)

```

++3+-----+H+-----+H+-----+G+-----+I+-----+G+-----+I+-----+
++3+-----+G+-----+I+-----+G+-----+I+-----+G+-----+I+-----+

```

++4---+-----+H---+-----+G---+-----+I---+-----+G---+-----+I---+-----+7---
++4---+-----+G---+-----+I---+-----+G---+-----+I---+-----+5---+-----+I---+-----

3 Balls, Height 20

$(L, L_{\text{bound}}, \text{States}) = (1082, 1083, 1140)$

Type I Incomplete: (92 patterns)

inverses of:

```
I23131411E1111111411H131411E111111112E11411E111111A1111401
I22231411E111111181111F1411E111111112E11411E111111A1111401
H3131411E1111111411H131411E111111112E11411E111111A11116011
H2231411E111111181111F1411E111111112E11411E111111A11116011
G31411E11111117111G1411411E111111291111F11411E11111160111
F411711111E111F111111181111F1511161111F115111E12011161111
F411711111E111711112F15111E11111118111F015111E11111161111
F411711111E1112E111111181111F1511701111F115111E11111161111
F411711111E111720111G115111E111111181111E15111E11111161111
F3131D111111112D131E11111118111G0191111111C111E11111161111
F3131D12011111E1131E111111181111F191111111C111E11111161111
F9111111181111E131D11120111G113131D1111111C111E11111161111
F911111118111F0131D111111112F13131D1111111C111E11111161111
F3141F0111111181111F1318111111D112D1111111C111E11111161111
F9111111181111F1411E111111112F114116111F011411E11111161111
F9111111181111F1411E11120111G11141161111E11411E11111161111
F31411E111111181111F131811111E011E11111112B11E11111161111
F2318111111D112D1111111C111E111111911111E141F011111171111
F25111711111E15111E120111A11111E11711111E111F111111171111
D5111E111111A11111E11711111E1118120111G125111E11111171111
F51118201111F125111E111111A11111E11711111D111E11111171111
F231811111E011E11111112B11E111111911111E1411E11111171111
F291111111C111E111111911111E131D12011111E1131E11111171111
F291111111C111E11111191111F0131D111111112D131E11111171111
F25118011111E15111E111111A11111E11711111E1112E11111171111
G31318111111D112D1111111C111E1111115111G13141F01111115111
G22318111111D112D1111111C111E111111911111E141F01111115111
```

G315111611111F115111E1201115111G1411711111E111F11111115111
G225111711111E15111E120111A11111E11711111E111F11111115111
G313131D1111111C111E1111115111E131D11112011H11911111115111
G31411E11111112F114116111F011411E1111115111G1911111115111
G31411E11120111G114116111E11411E1111115111G1911111115111
G313131D1111111C111E111111511F0131D111111112G1911111115111
D5111E111111A11111E11711111E1118112011H1225111E11111115111
G251118201111F125111E111111A11111E11711111D111E11111115111
G3131811111E011E11111112B11E1111115111G131411E11111115111
G2231811111E011E11111112B11E111111911111E1411E11111115111
G31411E1111111711H01411411E111111A111112E1411E11111115111
G3191111111C111E1111115111G13131D12011111E1131E11111115111
G2291111111C111E111111911111E131D12011111E1131E11111115111
G2291111111C111E11111191111F0131D111111112D131E11111115111
G3191111111C111E111111511H013131D111111112D131E11111115111
G31511701111F115111E1111115111G1411711111E1112E11111115111
G225118011111E15111E111111A11111E11711111E1112E11111115111
G2411411E111111A111112E1411E1111118111G01411E11111116111
G411411E111111911112G1411411E1111115111G01411E11111116111
G2411411E111111291111F11411E111111901111F1411E11111116111
G411411E11111192011H11411411E1111115111F1411E11111116111
G411411E11111191111E11411E111111920111G12411E11111116111
G23131D11111112F1711111E11111D1111118111E0131D11111116111
G318111111D1111E1111115111G13131D12011111E1131D11111116111
G228111111D1111E111111911111E131D12011111E1131D11111116111
G23131D11120111G11711111E11111D11111181111D131D11111116111
G228111111D1111E11111191111F0131D11111112D131D11111116111
G318111111D1111E111111511H013131D11111112D131D11111116111
H22411411E111111A111112E1411E1111118111G01411E1111111411
H2411411E111111911112G1411411E1111115111G01411E1111111411
H31411411E111111A111112E1411E111111411I0131411E1111111411
H31411411E111111291111F11411E1111115011H131411E1111111411
H22411411E111111291111F11411E111111901111F1411E1111111411
H2411411E11111192011H11411411E1111115111F1411E1111111411
H2411411E11111191111E11411E111111920111G12411E1111111411
H223131D11111112F1711111E11111D1111118111E0131D1111111411
H313131D11111112H131711111E11111D11111141E0131D1111111411
H2318111111D1111E1111115111G13131D12011111E1131D1111111411
H2228111111D1111E111111911111E131D12011111E1131D1111111411
H223131D11120111G11711111E11111D11111181111D131D1111111411
H313131D111112011I1131711111E11111D111111411D131D1111111411
H2228111111D1111E11111191111F0131D11111112D131D1111111411
H2318111111D1111E111111511H013131D11111112D131D1111111411
H22225111E111111B11111D111E1201111F11118111111D18111111511


```

H22225111E111111B1111E0111E11111112E1118111111D18111111511
H3131411E1111111411H131411E11201111F111411E111111A11111511
H2231411E111111181111F1411E11201111F111411E111111A11111511
I222225111E111111B1111D111E1201111F11118111111D1811111131
I222225111E111111B1111E0111E11111112E1118111111D1811111131
I23131411E1111111411H131411E11201111F111411E111111A1111131
I22231411E111111181111F1411E11201111F111411E111111A1111131
I222225111E111111A1111E11711111D111E1201111F1111811111141
I222225111E111111A1111E1171111E0111E11111112E111811111141
I2222411E11111112F11411E11111119111F011411E11111191111141
I2222411E11120111G111411E11111119111E11411E11111191111141
J2222225111E111111A1111E1171111D111E1201111F111181111112
E25111E111111A1111E1171111D111E11111117111G0511181111112
E25111E111111A1111E1171111D111E1111111511H02511181111112
F25111E11111117111E05111E111111A1111E1171111E11181111112
G225111E1111111511E05111E111111A1111E1171111E11181111112
J2222225111E111111A1111E1171111E0111E11111112E11181111112
J22222411E11111112F11411E11111119111F011411E1111119111112
J22222411E11120111G111411E11111119111E11411E1111119111112
F2411E11111111611H0411411E11111119111E11411E1111119111112
F2411E1111111141I02411411E11111119111E11411E1111119111112

```

Type II Incomplete: (4 patterns, 4.3 days)

```

++4---+-----+I---+-----+I---+-----+H---+-----+J---+-----+C---+---+--
++4---+-----+H---+-----+J---+-----+H---+-----+J---+-----+C---+---+--
++5---+-----+I---+-----+H---+-----+J---+-----+C---+-----+J---+---+--
++5---+-----+H---+-----+J---+-----+G---+-----+J---+-----+J---+-----+

```

3 Balls, Height 21

(L, L_{bound}, States) = (1266, 1266, 1330)

Complete: (1 pattern, 29 minutes)

inverse of:

```
F511161111G11171111E111171111F171111A1111E1111111111711111
```

3 Balls, Height 22

$(L, L_{\text{bound}}, \text{States}) = (1469, 1470, 1540)$

Type I Incomplete: (at least 4 patterns -- calculation not finished)

inverses of:

```
I411411F1111111112I118111111511G011411F11111115111I1411411F11111115111
I411411F111112011J1181111115111F11411F11111115111I1411411F11111115111
I313131E111111112E131E11111111711J01318111111E11111G111111E1111115111
I313131E120111111F1131E111111117111I1318111111E11111G111111E1111115111
```

Type II patterns: calculation not performed

4 Balls

4 Balls, Height 5

$(L, L_{\text{bound}}, \text{States}) = (5, 5, 5)$

55550

4 Balls, Height 6

(L, L_{bound}, States) = (12, 12, 15)

Complete: (1 pattern)

+++1+---+1+4-

4 Balls, Height 7

(L, L_{bound}, States) = (30, 30, 35)

Complete: (1 pattern)

+++1+---+3+---+1+---+6+---+4--

4 Balls, Height 8

(L, L_{bound}, States) = (58, 60, 70)

Type I Incomplete: (26 patterns)

+++3+---+3+---+1+---+7+---+6+---+1+---+7+5+---+7+---
+++3+---+1+---+1+---+7-7+---+7+---+4+---+3+---+7+---
+++3+---+1+---+1+---+5+---+1+---+7+---+2+5+---+7+---
+++2+---+4+---+1+---+4+---+1+---+1+---+7+5+---+7+---
+++2+---+2+6+---+2+---+7+---+2+---+2+---+2+---+7+---
+++2+---+2+6+---+2+---+7+---+37+---+2+---+2+---+7+---
+++2+---+2+1+---+7+---+7+---+31+---+7+---+3+---+7+---
+++2+---+2+1+---+6+---+1+---+3+---+7+---+3+---+7+---
+++2+---+1+---+5+---+7+---+2+---+7+6+---+3+---+7+---
+++2+---+1+---+75+---+1+---+1+---+7+6+---+3+---+7+---
+++1+---+5+---+4+---+1+---+1+---+7+---+7+---+5+---

```

+++1+-----+5+-----+2--+6-----+2+-----+7+-----+2+-----+2+-----+5--
+++1+-----+5+-----+2+1+-----+7+-----+7+-----+31+-----+7+-----+6---
+++1+-----+5+-----+2+1+-----+6+-----+1+-----+3+-----+7+-----+6---
+++1+-----+5+-----+1+-----+75+-----+1+-----+1+-----+7+6+-----+6---
+++1+-----+5+-----+1+-----+5+-----+7+-----+2+-----+7+6+-----+6---
+++1+-----+3--+6+-----+1+-----+7+-----+3+-----+7+-----+7+-----+5---
+++1+-----+3--+1+-----+7+-----+7+-----+4+-----+7+-----+4+-----+6---
+++1+-----+3--+1+-----+7+-----+2+-----+1+-----+7+-----+5+-----+5---
+++1+-----+6+-----+6+-----+6+-----+6+-----+1+-----+6+-----+2--+6+-----+6---
+++1+-----+6+-----+6+-----+15+-----+1+-----+6+-----+2--+6+-----+6---
+++1+-----+6+-----+77+-----+6+-----+1+-----+6+-----+2--+6+-----+6---
++3+-----+6+-----+6+-----+1+-----+6+-----+2--+6+-----+3+-----+7---
++3+-----+6+-----+15+-----+1+-----+6+-----+2--+6+-----+3+-----+7---
++3+-----+77+-----+6+-----+1+-----+6+-----+2--+6+-----+3+-----+7---

```

Type II Incomplete: (18 patterns)

```

+++3+-----+3+-----+1+-----+7+-----+6+-----+1+17+--7+5+-----+7---
+++3+-----+1+-----+1+-----+7+2+--+4+-----+1+-----+7-7+-----+7---
+++3+-----+1+-----+1+17+--7+2+--+4+-----+1+-----+7-7+-----+7---
+++3+-----+1+-----+1+17+--7-7+-----+7+-----+4+-----+3+-----+7---
+++3+-----+1+-----+1+17+--5+-----+1+-----+7+-----+2--+5+-----+7---
+++2+-----+4+-----+1+-----+4+-----+1+-----+1+17+--7+5+-----+7---
+++2+-----+2--+1+--+17-6+-----+1+-----+3+-----+7+-----+3+-----+7---
+++2+-----+1+-----+5+-----+7+-----+2+17+--7+6+-----+3+-----+7---
+++1+-----+5+-----+4+-----+1+-----+1+1+--+17-7+-----+7+-----+5---
+++1+-----+5+-----+2--+6+-----+2+-----+7+-----+2+-----+11+--+5---
+++1+-----+5+-----+2--+6+-----+2+-----+7+-----+37+-----+2+-----+5---
+++1+-----+5+-----+2--+1+--+17-6+-----+1+-----+3+-----+7+-----+6---
+++1+-----+5+-----+1+-----+5+-----+7+-----+2+17+--7+6+-----+6---
+++1+-----+3--+6+-----+1+-----+7+-----+3+--+17-7+-----+7+-----+5---
+++1+-----+3--+1+--+17-7+-----+7+-----+4+-----+7+-----+4+-----+6---
+++1+-----+3--+1+--+17-7+-----+2+-----+1+-----+7+-----+5+-----+5---
+++1+-----+1+1+-----+7+-----+4--+6--+1+-----+7+-----+7+-----+5---
+++1+-----+1+1+-----+7+-----+4--+6--+1+--+17-7+-----+7+-----+5---

```

4 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (112, 112, 126)$

Complete: (5 patterns)

```
+++4---+1+---+5-+---+8-+---+5-+---+1+---+8-+---+5-+---+1+---+8-+---+2-+---+8-+---+6-+---+8-+---
+++3---+1+---+7-+---+1+---+8-+---+7-+---+7-+---+1+---+7-+---+2-+---+6-+---+8-+---+4-+---+8-+---
+++2---+5-+---+1+---+8-+---+4-+---+8-+---+2-+---+8-+---+6-+---+1+---+3-+---+8-+---+6-+---+8-+---
+++2---+1+---+7-+---+7-+---+1+---+8-+---+5-+---+8-+---+4-+---+2-+---+8-+---+3-+---+6-+---+8-+---
+++1+---+6-+---+2-+---+6-+---+8-+---+2-+---+8-+---+2-+---+7-+---+1+---+8-+---+5-+---+8-+---+6-+---
```

4 Balls, Height 10

(L, L_{bound}, States) = (188, 188, 210)

Complete: (9 patterns, 11 hours)

```
+++4---+1+---+3-+---+9-+---+9-+---+3-+---+9-+---+9-+---+3-+---+2-+---+9-+---+2-+---+9-+---+
+++3---+1+---+7-+---+9-+---+2-+---+9-+---+8-+---+1+---+9-+---+4-+---+9-+---+2-+---+9-+---+
+++2---+4-+---+8-+---+1+---+9-+---+9-+---+6-+---+1+---+9-+---+6-+---+9-+---+2-+---+1+---+
+++2---+4-+---+7-+---+9-+---+4-+---+9-+---+4-+---+1+---+9-+---+8-+---+1+---+8-+---+7-+---+
+++2---+4-+---+7-+---+9-+---+4-+---+9-+---+4-+---+1+---+9-+---+8-+---+1+---+6-+---+9-+---+
+++2---+4-+---+7-+---+9-+---+2-+---+9-+---+7-+---+1+---+3-+---+9-+---+9-+---+3-+---+9-+---+
+++2---+4-+---+7-+---+9-+---+2-+---+9-+---+6-+---+1+---+9-+---+6-+---+9-+---+2-+---+1+---+
+++1+---+7-+---+4-+---+8-+---+1+---+9-+---+9-+---+6-+---+1+---+9-+---+6-+---+9-+---+2-+---+
+++1+---+7-+---+4-+---+7-+---+9-+---+2-+---+9-+---+6-+---+1+---+9-+---+4-+---+9-+---+2-+---+
```

4 Balls, Height 11

(L, L_{bound}, States) = (300, 300, 330)

Complete: (144 patterns)

inverses of:

5A2681117182A1229A1112A1283131

5A27319113A22A312A129128116151
5A415A118131719314A15191811611
5A415A118131814A15191319711611
5A51419A1122A14173191491811611
5A518131814A12A231718118911511
6A224A12A12911711818312A273131
6A2284118182A11512A612A1711611
6A24172A114A12A12911315A163141
6A241731914A129128112A61811611
6A24192911315A14812A114A161141
6A2613188131913A26118191811611
6A2713181929118112A414A1261611
6A27131914A131929118112A246131
6A313A228131619618118191811611
6A3192316181727117A12A12911511
6A8112A114A125161619419A111511
6A81131814A12A2A111317A1261611
74A12931911418A171117199112611
788111618A1317181189112A126411
797111619293181181913192812611
7A22A12911319512A318312A163141
7A22A13191611713A239312A171231
7A22A13191611713A7122A12831611
7A31413A259112A17131914A171131
7A31418911316A17131914A1711611
7A31419A1122A12841161791811611
7A318131616A12A231718118911511
7A318191113A2A4112A12A23171251
7A319122A128411713A612A1711611
7A319122A151918118113A23841151
7A3191318191811811771161926411
83A319122A151918118113A2741151
8491319A1131912971116199131611
884116194182A11518191319711611
884117192A12911319418318171611
884117193183181717118A12911511
88411814A151918118113A51912511
884118183181717118A14118911511
89312A151916118194113A51812611
8931318A13191611719314A1712611
8931711618A13191319314A1712611
893181181913192297111619712611
893181181916118194116711881411
8931911418A12611719314A1712611

8931911418A14118611814A1712611
894118141941929117117192812611
899111161829A1112A151299112611
8A2A1112A1413A81131617A1261611
8A2A11131841991131814A15171611
8A3131929128116161951719117411
8A313193193151619A131291812611
8A3131971171189128118151817411
8A3181171189128118151719317411
8A318118113A518131814A15181511
8A318118113A519122A15151817411
8A51113A248131819A1112A1517131
93491319A113191297111619913131
936281189112912931419A12811151
938317118912711819319131961141
938318113A519122A1419319161151
9383181172A1418A111418A1261151
9384116194182A1151819131971131
9384117192A1291131941831817131
939312A151916118194113A5181231
93931318A13191611719314A171231
93931711618A13191319314A171231
939318118191319229711161971231
93931911418A12611719314A171231
93931911418A14118611814A171231
9394112A612A128129113A26171151
939411814194192911711719281231
9399111161829A1112A15129911231
93A2A1112A1413A81131617A126131
93A2A11131841991131814A1517131
93A313193193151619A13129181231
94418911811814197131913A271131
94418911811814197131913A811131
948128118911319419315191941141
948128118911711819314192941141
948131914A1313A2618112A6181131
949123192A31319611817191173151
949123194193191319611619183131
949129114194192319611619183131
94A129113192841171931831817131
94A129117117193183181419283131
94A131419283181189117118183141
94A131519183181189113194183141
94A131718118911319284118183141

94A141189113192841181831817131
94A141189113194183181519183131
94A141189117118183181419283131
952A128129113A7117116192941141
95611818A111418A141172A1283131
9581171188131914A1313A71181141
959112A17131713A413A22A1317131
95911318619113A27131914A161141
95911711617A128129113A22941141
959118112A414A12619113A2713151
95911811812A313A27117118813151
95911811812A313A81117118813151
962A114193193191611616A1283131
96319141891131941731914A171131
963191419319291131951519183131
964117A12416A11631819291181141
964118419319291181181519183131
9681117182A1229A111316A1283131
968111719229291181181419283131
97319113A224A129128112A6161151
97319114A12A51113A519129115151
9741161925A1141931831819181131
97411713A512A2A1122A1249181131
97411813A224A129128112A6181131
97411818319141891131924A171131
992411819231929129114192941141
992411819319131929123192941141
9A251115914A191129113A71181141
9A3128118191319319611159183131
9A411811812A81181115913A271131
9A411811812A81181115913A811131
A336281189112912931419A1281112
A338318113A519122A141931916112
A3383181172A1418A111418A126112
A3384117193182A115181918117113
A3384118182A115181913197117113
A3394112A612A128129113A2617112
A33A318118117A118131814A151413
A349123192A3131961181719117312
A34A12611817281172A1292A111413
A358118117A118131814A12A411413
A359112A17131716A113A22A131413
A359118112A414A12619113A271312
A35911811812A313A2711711881312

A35911811812A313A8111711881312
A37319113A224A129128112A616112
A37319113A22A312A1291281161613
A37319114A12A51113A51912911512
A39313191931811819131931961113
A4294112A252A128129113A7117112
A42A41171911791113A24912811612
A42A41171911791113A262A1151612
A44182A115181918118113A2384112
A4A1312918122A13A3131931931413
A529128112A238319113A251916112

4 Balls, Height 12

$(L, L_{\text{bound}}, \text{States}) = (452, 452, 495)$

Complete: (45 patterns, 97 minutes)

inverses of:

8B22313B7112B12B12A11131A7131918131A1731611
8B22319B11131A2B14118131A7131918131A1731611
8B23191231A29241171A2A141181A1319B111811611
8B23191231A292411819B11131A2A141181A1711611
93A2B11131A2A141181A131A3191811161AA1231611
94B12319B11131A61171A118411616B1712B1171411
9728111891911181A17119119B11161171A31931511
9741161A51913181A17119119B11131A315B1171411
9741161A713181A1319B1118118117B112B23181411
9741191681117A17119119B11131AB1141191912511
99241161A519131A151A17119119B11131A31931511
9A23131A713181A131A3141A2B12B11161191822611
9A31911181A12812A11841171A5119B111511961511
9B22B11131A611418B12314B1712B11713191831611
9B41131A2A14119161A11823181A2B113141A712511
9B41131A318317119B141181A1191316181A2911411
9B41131A318317119B141181A1191316181AA111411
9B4118112B173131A4182A1118B11141A251A161141

9B411811911931A15117A11914119A1411619182411
A33A2B11131A2A141181A131A3191811161AA123131
A3728111891911181A17119119B11131A3193171151
A39231A114A1291271171A5119B1115119A1A11141
A39231A11571171A5119B1112A12914119A1A11141
A39231A1193171171A5119B111511951A1291911141
A394119119B1112A1291231A41841171A2A1A11141
A3A23131A713181A131A3141A2B12B1116119182231
A3B22B1112A1291231A4183191173171A2A1A11141
A3B22B11131A611418B12314B1712B1171319183131
A4B12B1116119193191141A29231811931A31913141
A531A3191291141A41A2B11131A2A141161A1923131
A57119119B11131A3193191151A1728111891911141
A5B113171A11823181A3191231A2B41131A2A141141
A6B11131A3193131A29241181A319131A151A171131
A6B11131A31A411418B12314B1712B117131A173131
A6B11131A611418B12314B12A312B11712A11841151
A73131A2A313181A131A3141A2B12B1116119182231
A73191141A41A2B11131A241A2291911161A1923131
A73191141A7131A1419B11131A318231A14B1271131
A731A11914117B122B3161161A172A1113B71181131
A8241171A2A14119191291141A23B22319B11181131
A82411819B11131A2A14119191291141A23B2271131
B33728111891911181A17119119B11131A319317112
B36B11131A611418B12314B12A312B11712A1184112
B42B41131A3182A1118B11151A151A17112B1731313
B52A1A11112B2392319113B241A2B1112A129127112

4 Balls, Height 13

$(L, L_{\text{bound}}, \text{States}) = (660, 660, 715)$

Complete: (at least 245 patterns -- calculation not finished)

partial list -- inverses of:

8A8111171BB11411911A318181119C131B11A11181181B1B1171111
5C8141191B11711811A2C1411A17111915C11A131A1114C18171111

5C51A1411811A2C131B11711191A41191911A1181A1114C18171111
8A8111181AC1119111711AA131911192C1711191911811A1A171111
882A11199111811AC118111161B515191B11A112A1151B1B1271111
8C312B131B11A11911191B1811191193141BB12511171B2B1271111
8C312B131B11A11811171B2B1511191B1811191193141BB13181111
5C515191811191B31B11A11191191811811A811A1114C1519181111
6C7119112A114C12C1312C11511A3191811161BC151112A19181111
8A8111171B261B11A11911181B1711A1181A1119911171199181111
6C261711911AC1112C11711811A1A128111617C125181B111881111
6C711911A1114C151A128111617C12C1312C11711A1161A11881111

5 Balls

5 Balls, Height 6

$(L, L_{\text{bound}}, \text{States}) = (6, 6, 6)$

666660

5 Balls, Height 7

$(L, L_{\text{bound}}, \text{States}) = (18, 18, 21)$

Complete: (1 pattern)

++++1+----1+---+4-

5 Balls, Height 8

$(L, L_{\text{bound}}, \text{States}) = (49, 49, 56)$

Complete: (3 patterns)

```
++++2--++++1+--++1+--++6--++2--++2--++7--  
++++1+--++4--++1+--++1+--++6--++2--++6--  
++++1+--++3--++1+--++6--++1+--++3--++6--
```

5 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (112, 112, 126)$

Complete: (5 patterns)

```
++++3--++++1+--++4--++1+--++8--++2--++7--++1+--++7--++1+--++3--++7--++3--++8--  
++++1+--++5--++1+--++3--++7--++2--++8--++2--++2--++1+--++8--++2--++8--++6--  
++++1+--++3--++6--++1+--++7--++5--++1+--++4--++1+--++8--++2--++2--++8--++7--  
++++1+--++3--++1+--++7--++1+--++8--++4--++1+--++8--++4--++1+--++4--++8--++5--  
++++1+--++3--++1+--++6--++8--++3--++1+--++7--++1+--++5--++1+--++8--++4--++7--
```

5 Balls, Height 10

$(L, L_{\text{bound}}, \text{States}) = (225, 226, 252)$

Type I Incomplete: (752 patterns, 4.5 minutes)

partial list -- inverses of:

9939229139931392939318414

9939229139481392939318414
9933931892292681293958113
9933922962916292939318414
9933922939248188129295314
9933891139393491691297313
9933891139393491397291613
9793181841882491494918313
9793171691494913991296114
9793129629282491494918313
9792911393697117913897112
9792911393692617913897112
9792281395813924939292612
9792271881491493929318414
9792271691692293929318414

993922913993139293931A71414
993922913993139293931895014
993922913948139293931A71414
993922913948139293931895014
993393189A11292681293958113
993393189293092681293958113
993392296291629293931A71414
993392296291629293931895014
99339229392481A718129295314
993392293924818990129295314
9793181A7141882491494918313
979318189501882491494918313
9793129A5129282491494918313
979312969309282491494918313
979228139581392493929A11612

Type II Incomplete: (86 patterns, 28 minutes)

++++4---+---+1+---+---+7+---+---+5---+---+1+---+---+1+19---+---+9---+---+6---+---+9---+---+3---+---+1+---+---+9---+---+
++++4---+---+1+---+---+7+---+---+2+---+---+7+---+---+2+---+---+1+---+---+19-9---+---+9---+---+3---+---+1+---+---+9---+---+
++++4---+---+1+---+---+7+---+---+1+---+---+1+---+---+8+---+---+8+---+---+8+---+---+3---+---+1+---+---+9---+---+7+---+---+
++++4---+---+1+---+---+5+---+---+1+---+---+9+---+---+7+---+---+1+---+---+1+19---+---+9---+---+8+---+---+3+---+---+8+---+---+
++++4---+---+1+---+---+5+---+---+1+---+---+9+---+---+6+---+---+1+---+---+9+---+---+7+---+---+1+---+---+3+---+---+7+---+---+
++++4---+---+1+---+---+1+---+---+8+---+---+6+---+---+1+---+---+9+---+---+2+---+---+9+---+---+7+---+---+1+---+---+8+---+---+
++++4---+---+1+---+---+1+---+---+6+---+---+9+---+---+7+---+---+1+---+---+8+---+---+2+---+---+6+---+---+1+---+---+9+---+---+
++++4---+---+1+---+---+1+---+---+6+---+---+9+---+---+7+---+---+1+---+---+8+---+---+1+---+---+5+---+---+1+---+---+19-9+---+---+
++++4---+---+1+---+---+1+---+---+6+---+---+9+---+---+7+---+---+1+---+---+8+---+---+1+---+---+5+---+---+1+---+---+19-9+---+---+
++++4---+---+1+---+---+1+---+---+6+---+---+9+---+---+2+---+---+1+---+---+19-9+---+---+9+---+---+3+---+---+1+---+---+9+---+---+

++++1+-----+5+-----+7+-----+3+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+2+-----+
++++1+-----+5+-----+7+-----+3+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+2+-----+
++++1+-----+5+-----+4+-----+2+-----+9+-----+8+-----+1+-----+5+-----+1+-----+9+-----+7+-----+1+-----+
++++1+-----+4+-----+7+-----+1+-----+1+19+-----+9+-----+8+-----+3+-----+8+-----+1+-----+9+-----+6+-----+
++++1+-----+4+-----+6+-----+1+-----+9+-----+7+-----+1+-----+4+-----+1+-----+19-9+-----+9+-----+7+-----+
++++1+-----+4+-----+6+-----+1+-----+9+-----+7+-----+1+-----+3+-----+7+-----+9+-----+4+-----+1+-----+
++++1+-----+4+-----+6+-----+1+-----+9+-----+4+-----+1+-----+9+-----+5+-----+9+-----+3+-----+1+-----+
++++1+-----+4+-----+2+-----+8+-----+1+-----+9+-----+4+-----+9+-----+3+-----+1+-----+9+-----+7+-----+
++++1+-----+4+-----+2+-----+8+-----+1+-----+9+-----+4+-----+9+-----+3+-----+1+-----+9+-----+7+-----+
++++1+-----+4+-----+2+-----+7+-----+9+-----+3+-----+1+-----+9+-----+4+-----+1+-----+9+-----+8+-----+
++++1+-----+4+-----+2+-----+7+-----+9+-----+3+-----+1+-----+9+-----+4+-----+1+-----+9+-----+8+-----+
++++1+-----+3+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+6+-----+1+-----+
++++1+-----+3+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+6+-----+1+-----+
++++1+-----+3+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+6+-----+1+-----+
++++1+-----+3+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+5+-----+1+-----+9+-----+6+-----+1+-----+
++++1+-----+3+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+5+-----+1+-----+9+-----+6+-----+1+-----+
++++1+-----+3+-----+7+-----+1+-----+8+-----+1+-----+9+-----+6+-----+1+-----+9+-----+4+-----+8+-----+
++++1+-----+3+-----+7+-----+1+-----+8+-----+1+-----+9+-----+6+-----+1+-----+9+-----+4+-----+3+-----+
++++1+-----+3+-----+7+-----+1+-----+7+-----+9+-----+6+-----+1+-----+7+-----+5+-----+1+-----+1+19+-----+
++++1+-----+3+-----+7+-----+1+-----+7+-----+9+-----+6+-----+1+-----+5+-----+1+-----+9+-----+6+-----+
++++1+-----+3+-----+7+-----+1+-----+7+-----+9+-----+2+-----+6+-----+1+-----+9+-----+4+-----+1+-----+
++++1+-----+3+-----+7+-----+1+-----+7+-----+9+-----+2+-----+6+-----+1+-----+9+-----+4+-----+1+-----+
++++1+-----+3+-----+6+-----+2+-----+9+-----+3+-----+9+-----+4+-----+1+-----+9+-----+6+-----+1+-----+
++++1+-----+3+-----+6+-----+1+-----+9+-----+2+-----+9+-----+7+-----+1+-----+8+-----+1+-----+7+-----+
++++1+-----+3+-----+1+-----+8+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+4+-----+1+-----+
++++1+-----+3+-----+1+-----+8+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+4+-----+1+-----+
++++1+-----+3+-----+1+-----+7+-----+9+-----+3+-----+9+-----+4+-----+1+-----+9+-----+6+-----+1+-----+
++++1+-----+1+-----+7+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+1+-----+
++++1+-----+1+-----+7+-----+7+-----+2+-----+1+-----+19-9+-----+9+-----+7+-----+1+-----+7+-----+1+-----+

5 Balls, Height 11

(L, L_{bound}, States) = (420, 420, 462)

Complete: (59346 patterns, 30 hours)

partial list -- inverses of:

```
6A3A52A12A2929127199137A114A3AA13191913561
6A5618A12A292912A139A27118188A11813AA16411
6A5A1722A14A3A2418A161918A122A39413A718241
7A62A163193842A2942A2A139318A1619199181161
9A531881915199619118A2A1519182A318A2711661
A4A429318A17181918812719923A22A53197A11613
A6A22A711813A394A12619913A3194A123A3951712
A73A319418A14174A1934A12A82317A17A12A16314
A83A222A3A2529182A22A3A3192A1712A2A4415714
A8A2318184199413A248A222A4A13A22A2A2A17112
```

6 Balls

6 Balls, Height 7

$(L, L_{\text{bound}}, \text{States}) = (7, 7, 7)$

7777770

6 Balls, Height 8

$(L, L_{\text{bound}}, \text{States}) = (24, 24, 28)$

Complete: (1 pattern)

+++++1+-++++1+-++++1++5-

6 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (74, 74, 84)$

Complete: (1 pattern)

+++++1+--++++3-+-++++1+---++1+1+++--+8-+++++2-++++-+1+-----8-+-++++4--

6 Balls, Height 10

$(L, L_{\text{bound}}, \text{States}) = (188, 188, 210)$

Complete: (9 patterns) dual of (4,10) above

6 Balls, Height 11

$(L, L_{\text{bound}}, \text{States}) = (420, 420, 462)$

Complete: (59346 patterns) dual of (5,11) above

7 Balls

7 Balls, Height 8

$(L, L_{\text{bound}}, \text{States}) = (8, 8, 8)$

88888880

7 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (32, 32, 36)$

Complete: (1 pattern)

+++++1+-+++++1+-+++++1+-+++++5-

7 Balls, Height 10

$(L, L_{\text{bound}}, \text{States}) = (108, 108, 120)$

Complete: (1 pattern)

+++++1+-+++++1+-+++++4-+++++1+-+++++1+-+++++8-+++++1+-+++++6-+++++1+-+++++1+-+++++4-+++++7--

7 Balls, Height 11

$(L, L_{\text{bound}}, \text{States}) = (300, 300, 330)$

Complete: (144 patterns) dual of (4,11) above

8 Balls

8 Balls, Height 9

$(L, L_{\text{bound}}, \text{States}) = (9, 9, 9)$

999999990

8 Balls, Height 10

$(L, L_{\text{bound}}, \text{States}) = (40, 40, 45)$

Complete: (1 pattern)

++++++1+-+++++1+-+++++1+-+++++1+-+++++1+-++++6-

8 Balls, Height 11

$(L, L_{\text{bound}}, \text{States}) = (149, 150, 165)$

Type I Incomplete: (18 patterns) dual of (3,11) above

Type II Incomplete: (0 patterns)

8 Balls, Height 12

(L, L_{bound}, States) = (452, 452, 495)

Complete: (45 patterns) dual of (4,12) above

9 Balls

9 Balls, Height 10

(L, L_{bound}, States) = (10, 10, 10)

AAAAAAAAA0

9 Balls, Height 11

(L, L_{bound}, States) = (50, 50, 55)

Complete: (1 pattern)

+++++++1+-+++++++1+-+++++++1+-+++++++1++++-++++6-

9 Balls, Height 12

(L, L_{bound}, States) = (200, 201, 220)

Type I Incomplete: (28 patterns, 3.2 hours) dual of (3,12) above

Type II Incomplete: (2 patterns, 4.8 hours) dual of (3,12) above

9 Balls, Height 13

(L, L_{bound}, States) = (660, 660, 715)

Complete: (at least 245 patterns -- calculation not finished) dual of (4,13) above
